

$$\sin^5 x + \cos^5 x = 1 \text{ решить разложив по } a^5 + b^5 = \dots$$

$$a^5 + b^5 = (a+b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$$

$$\sin^5 x + \cos^5 x = (\sin x + \cos x)(\sin^4 x - \sin^3 x \cos x + \sin^2 x \cos^2 x - \sin x \cos^3 x + \cos^4 x)$$

$$(\sin x + \cos x)(\sin^4 x - \sin^3 x \cos x + \sin^2 x \cos^2 x - \sin x \cos^3 x + \cos^4 x) = 1$$

$$(\sin x + \cos x)(\sin^4 x + 2\sin^2 x \cos^2 x + \cos^4 x - \sin^3 x \cos x + \sin^2 x \cos^2 x - \sin x \cos^3 x - 2\sin^2 x \cos^2 x) = 1$$

$$(\sin x + \cos x)((\sin^2 x + \cos^2 x)^2 - \sin^3 x \cos x - \sin x \cos^3 x - \sin^2 x \cos^2 x) = 1$$

$$(\sin x + \cos x)(1 - \sin^3 x \cos x - \sin x \cos^3 x - \sin^2 x \cos^2 x) = 1$$

$$(\sin x + \cos x)(1 - \sin x \cos x (\sin^2 x + \cos^2 x) - \sin^2 x \cos^2 x) = 1$$

$$(\sin x + \cos x)(1 - \sin x \cos x - \sin^2 x \cos^2 x) = 1$$

$$y = \sin x + \cos x$$

$$t = \sin x \cos x$$

Замена $y = \sin x + \cos x$

$$y^2 = (\sin x + \cos x)^2 = \sin^2 x + 2\sin x \cos x + \cos^2 x$$

$$y^2 = \sin^2 x + 2\sin x \cos x + \cos^2 x$$

$$y^2 = 1 + 2\sin x \cos x$$

$$\sin x \cos x = (y^2 - 1)/2$$

$$y(1 - (y^2 - 1)/2 - (y^2 - 1)^2/4) = 1$$

$$y(1 - (2(y^2 - 1) + (y^2 - 1)^2)/4) = 1 \quad | \cdot 4$$

$$y(4 - 2(y^2 - 1) - (y^2 - 1)^2) = 4$$

$$4y - 2y(y^2 - 1) - y(y^4 - 2y^2 + 1) = 4$$

$$4y - 2y^3 + 2y - y^5 + 2y^3 - y = 4$$

$$6y - 2y^3 - y^5 + 2y^3 - y = 4$$

$$5y - y^5 = 4$$

$$y^5 - 5y + 4 = 0$$

$$y = 1$$

$$\sin x + \cos x = 1$$

$$\sqrt{2}(\sin x/\sqrt{2} + \cos x/\sqrt{2}) = 1$$

$$1/\sqrt{2} = \sin t$$

$$1/\sqrt{2} = \cos t$$

$$t = \pi/4$$

$$\sqrt{2}(\sin x \cos t - \cos x \sin t) = 1$$

$$\sqrt{2} \sin(x + \pi/4) = 1$$

$$\sin(x + \pi/4) = 1/\sqrt{2}$$

$$x + \pi/4 = \pi/4 + 2\pi n$$

$$x = 2\pi n$$

$$x + \pi/4 = 3\pi/4 + 2\pi n$$

$$x = \pi/2 + 2\pi n$$

$$\text{Ответ: } 2\pi n; \pi/2 + 2\pi n$$

	1	0	0	0	-5	4
1	1	1	1	1	-4	0
1	1	2	3	4	0	

$$y^4 + y^3 + y^2 + y - 4 = 0$$

$$y = 1$$

$$1 \cdot y^3 + 2y^2 + 3y + 4 = 0$$

+ - делитель 4 / делитель 1

+ -2/1

+ -4/1

+ -1/1

$$\sin^{2003} x + \cos^{2003} x = 1$$

Случай 1:

$$\sin x = 1$$

$$x = \pi/2 + 2\pi n$$

$$\cos x = 0 \text{ при } \pi/2 + 2\pi n$$

Случай 2:

$$\cos x = 1$$

$$x = 2\pi n$$

$$\sin x = 0 \text{ при } 2\pi n$$

Случай 3

$$\sin x \neq 1 \ \&\& \ \cos x \neq 1$$

ТОЖДЕСТВО

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^{2003} x + \cos^{2003} x < 1$$

$$\text{Ответ: } 2\pi n; \pi/2 + 2\pi n$$